Cleaning Correlation Matrices from Random Noise

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Portfolio problem

- How to construct the "optimal" portfolio out of N, possibly correlated instruments?
  - optimal return?
  - optimal risk (VAR)?
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- It is important to know the behaviour of the instruments in the past: data collection.
- Past returns do not tell us the expected return (autocorrealtion $\sim$ 15m.
- Past volatilities are rather stable and have predictive power to the future values ("adiabatic process").

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Covariance matrix

- The basic object of the portfolio analysis is the Covariance Matrix.

$$C_{ij} = \langle x_i(t) x_j(t) \rangle_t = \frac{1}{T} \sum_{t=1}^{T} x_i(t) x_j(t).$$

with price changes

$$x(t) = \log \frac{P(t)}{P(t-1)} - Div(t).$$

- Goal: minimize risk of the portfolio (with a gaussian assumption)

$$R = \langle p_i C_{ij} p_j \rangle$$

with portfolio entries p_i's.

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Markowitz solution

- Markowitz's solution to the problem (without considering return information)

$$p_i = \frac{\sum_k C_{ik}^{-1}}{\sum_{i,k} C_{ik}^{-1}} \qquad \text{(Markowitz)}$$

with a minimal risk

$$R_{min} = \frac{1}{\sum_{i,j} C_{ij}^{-1}}$$

- Inclusion of returns to the minimization procedure increases risk.

- In case of non-Gaussian processes the form (definition) of the $\mathbf{C}$ covariance matrix is extremely important, and has an effect on the optimization procedure.

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Noise

- The price-change series matrix typically is of size $N \times T$, timeseries of length T for N instruments,

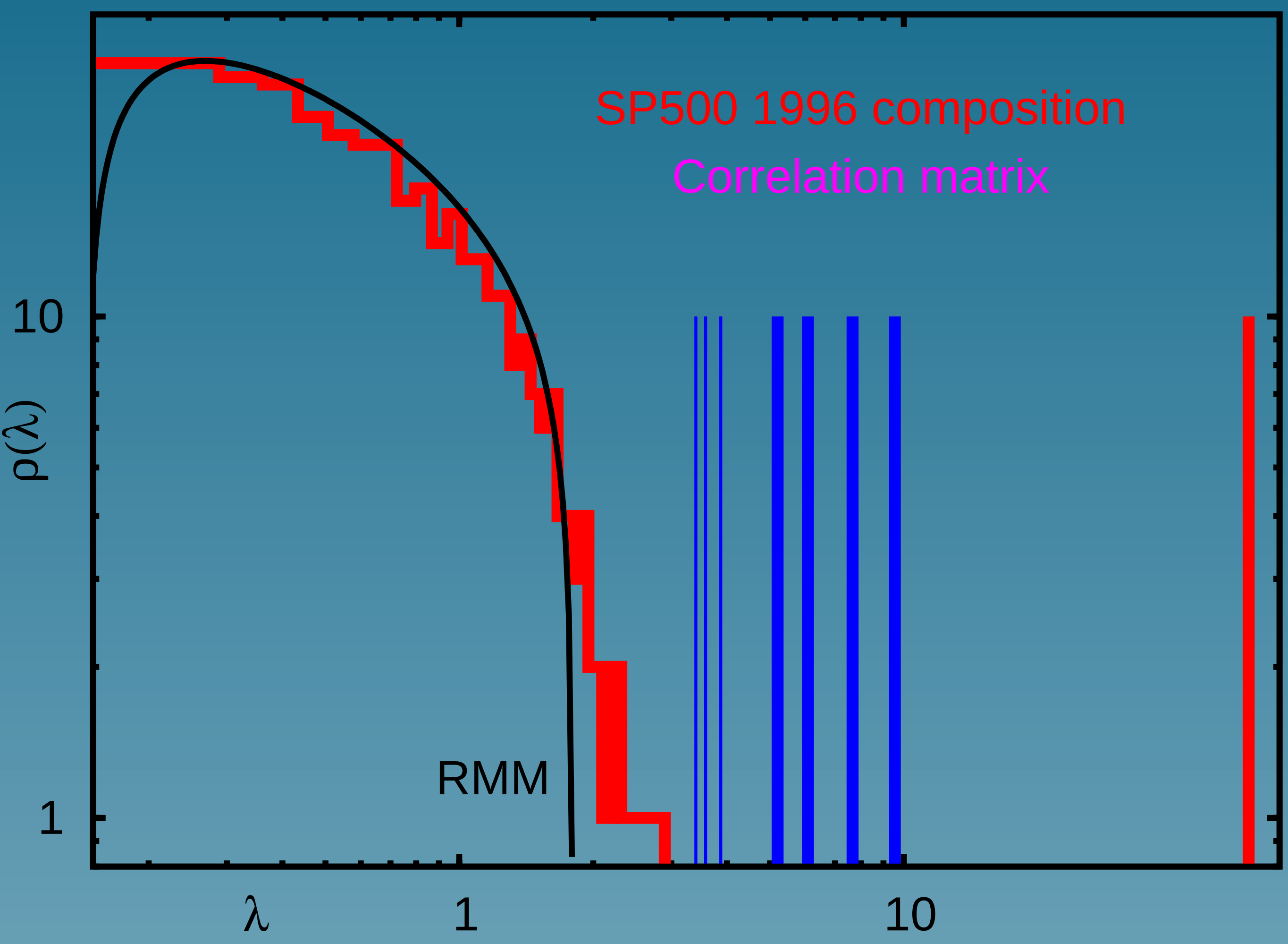
$$\mathbf{A} = \underbrace{\begin{pmatrix} x_1(1) & x_1(2) & \ldots & x_1(T) \\ x_2(1) & x_2(2) & \ldots & x_2(T) \\ \vdots & & \ddots & \\ x_N(1) & x_N(2) & \ldots & x_N(T) \end{pmatrix}}_{T} \left.\vphantom{\begin{pmatrix} x \\ x \\ x \\ x \end{pmatrix}}\right\} N$$

- The Covariance matrix is $\mathbf{C} = \frac{1}{T}\mathbf{A}\mathbf{A}^T$.

- Entries $x_i(t)$ are usually correlated, and

- are dressed with NOISE, hence the Covariance matrix is also NOISY.

Spectrum of the Correlation Matrix



RMM sectors market

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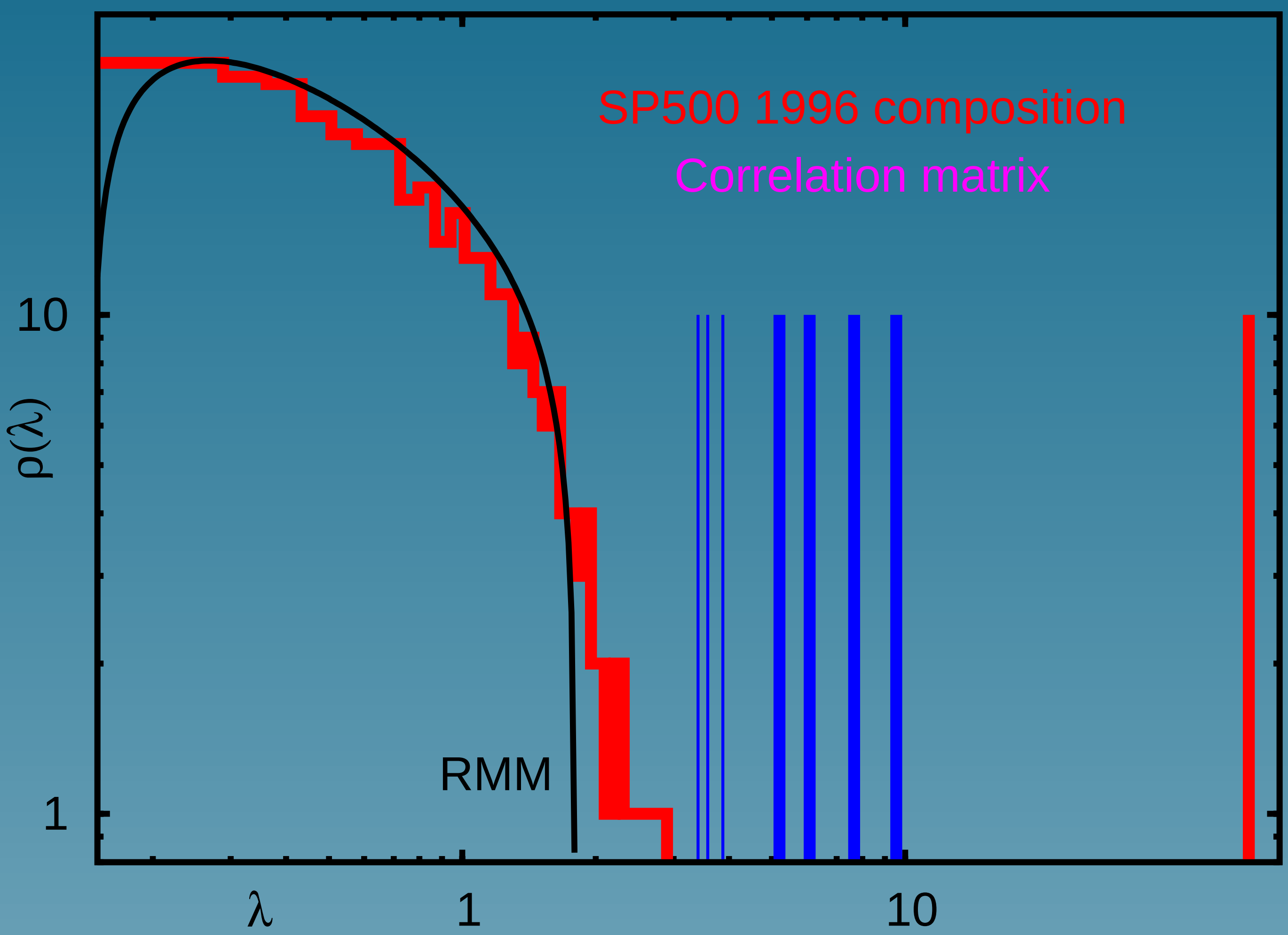

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Wishart matrices

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- spectrum (with $r = N/T$, $\mathbf{M}$ being random gaussian)

$$\varrho(\lambda) = \frac{1}{2r\pi\lambda}\sqrt{(\lambda - \lambda_{min})(\lambda_{max} - \lambda)},$$

$$\lambda_{min,max} = (1 \pm \sqrt{r})^2$$

- if $r > 1$ there are additional zero modes ($r\delta_0$).
- for $N/T \to 0$ only the "real" correlation survives at $\lambda = \mathbf{1}$.

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How to deal with Random Matrices?

- D.V. Voiculescu, *Invent. Math.* **104** (1991) 201.
- Free Random Variables
- Eg.: Gaussian distribution (numbers)

$$p(x) = \frac{1}{\sqrt{2\pi\sigma}}\, e^{-\frac{x^2}{2\sigma^2}}$$

- What is the distribution for $x_1 + x_2$, both Gaussian?

$$p_2(S = x_1 + x_2) = p(x_1) \otimes p(x_2), \quad \text{or}$$

$$\varphi_2(q) = \varphi(q)\varphi(q), \quad \text{with} \qquad \varphi(q) = \int dx\, e^{iqx}\; p(x), \text{or}$$

$$\ln \varphi_2(q) = \ln \varphi(q) + \ln \varphi(q)$$ ADDITION LAW

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  with $V(M) = \frac{1}{2}\,\mathrm{Tr}\; M^2$.

- for Gaussian matrices it fulfilles a simple algebraic eq:

$$G(z) = (z - G(z))^{-1} \quad \rightarrow \quad G(z) = \frac{1}{2}\left(z - \sqrt{z^2 - 4}\right)$$

and spectral density $\varrho(\lambda) = -\frac{1}{\pi}\lim_{\epsilon\to 0}\,\mathrm{Im}\; G(z)\big|_{z=\lambda+i\epsilon}$,

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Addition Law

- Condsider random matrices M_1 and M_2
- $[M_1, M_2] \neq 0!$ do not commute
- What is the spectrum of $M_1 + M_2$ on ensemle average?

$$G_{1+2}(z) = \int dM_1 dM_2 \, e^{-NV(M_1)} \, e^{-NV(M_2)} \frac{1}{N} \mathrm{Tr}_N \frac{1}{z - M_1 - M_2}$$

- Define $R(z)$ as $G[R(z) + 1/z] = z$, and R transform is ADDITIVE for matrix addition:

$$R_{1+2}(z) = R_1(z) + R_2(z)$$

- Eg.: add two matrices from the same Gaussian distribution:

$$R_1(z) = R_2(z) = z \to R_{1+2}(z) = 2z \to G(z) = \frac{1}{4}\left(z - \sqrt{z^2 - 8}\right)$$

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and the resulting distribution is the rescaling by $\sqrt{2}$:

$$\varrho(\lambda) = \frac{1}{4\pi}\sqrt{8-\lambda^2}$$

Multiplication Law

- What is the spectrum of $M_1 \cdot M_2$ on ensemle average? (positive definite, symmetric matrices)
- Define $S(z)$ as

$$S(z) = \frac{1+z}{z}\chi(z), \qquad \frac{1}{\chi(z)}G[\frac{1}{\chi(z)}] - 1 = M[\frac{1}{\chi(z)}] = z$$

where M is the generating function.

- $S(z)$ is MULTIPLICATIVE

$$S_{1\cdot 2} = S_1(z) \cdot S_2(z)$$

Multiplication Law: Example

- consider an $N \times T$ matrix $\mathbf{X}$, and construct $\frac{1}{T}\mathbf{X}\mathbf{X}^{\mathbf{T}}$ (symmetric, positive definite matrix)

- Spectrum for the product of $\mathbf{Y}$ $N \times N$ ($N >$T) matrices is known:

$$\varrho(z = \lambda^2) = \frac{\varrho_G(\lambda)}{\lambda} = \frac{1}{2\pi}\frac{\sqrt{4-z}}{\sqrt{z}}$$

- To get the proper spectrum of $\frac{1}{T}\mathbf{X}\mathbf{X}^{\mathbf{T}}$, $\frac{1}{T}\mathbf{Y}\mathbf{Y}^{\mathbf{T}}$ should be multiplied by the projector $\mathbf{P} = diag(1, ..., 1, 0, ...0)$:

$$S_{YY^T} = \frac{1}{1+z}, \qquad S_P = \frac{1+z}{m+z}$$

with $m = T/N$. Finally, with the inverse transformation one recovers

$$\varrho(\lambda) = (1-m)\delta(\lambda)\, \tfrac{1}{2\pi}\sqrt{(\lambda_> - \lambda)(\lambda - \lambda_<)}$$

with $\lambda_{<>} = (1 \pm \sqrt{m})^2$ (Marchenko-Pastur result).

Multiplication Law: Finances

- Let $\mathbf{C}$ be the spatial (instrument) correlation, $\mathbf{A}$ the temporal correlation. Constructing the *empirical* correlation matrix

$$G(z) = \frac{1}{N}\left\langle \mathrm{Tr}_N \frac{1}{z - \frac{1}{T}\mathbf{X}\mathbf{X}^T} \right\rangle \quad \text{with measure} \quad \int d\mathbf{X}\, e^{-\mathrm{Tr}_T \mathbf{X}^T \mathbf{A}^{-1} \mathbf{X}}$$

 one arrives at

$$M_{\mathbf{A}}(w) = m(z) \quad \text{with} \qquad w = \frac{z}{rm(z) M_{\mathbf{C}}^{-1}[rm(z)]}$$

 with $m(z)$ the generating function of the *empirical* matrix and $M_{\mathbf{A}}(z)$, $M_{\mathrm{C}}(z)$ the generating functions of true correlation matrices $\mathbf{A}$ and $\mathbf{C}$, respectively.

- The problem is INVERTIBLE! (Cleaning – Dressing)

Risks

- Clean system (Markowitz):

covariance matrix: $\mathbf{C}^{(0)}$,

portfolio entries: $$p_i^{(0)} = \frac{\sum_k {C^{(0)}}^{-1}_{ik}}{\sum_{i,k} {C^{(0)}_{ik}}^{-1}}.$$

- Noisy system:

covariance matrix: $\mathbf{C}$,

portfolio entries: $$p_i = \frac{\sum_k C^{-1}_{ik}}{\sum_{i,k} C^{-1}_{ik}}.$$

- Real risk ratio:

$$q_0^2 = \frac{\langle p|\mathbf{C}^{(0)}|p\rangle}{\langle p^{(0)}|\mathbf{C}^{(0)}|p^{(0)}\rangle}$$

Risks II

- This ratio can be computed analytically from Random Matrix Theory (gaussian case):

$$q_0^2 = \frac{\int d\lambda \, \frac{1}{\lambda^2} \, \varrho(\lambda)}{\left(\int d\lambda \, \frac{1}{\lambda} \, \varrho(\lambda) \right)^2} \quad = \quad \frac{1}{1-r}$$

- For long enough time series r is small, and the optimal portfolio is fairly reconstructed from the noisy, empirical covariance matrix.

- For a large portfolio r is close to 1, and the optimization is completely meaningless!

- For $r > 1$ the system is completely unpredictable (the number of unknown parameters is larger than the number of measurements!)

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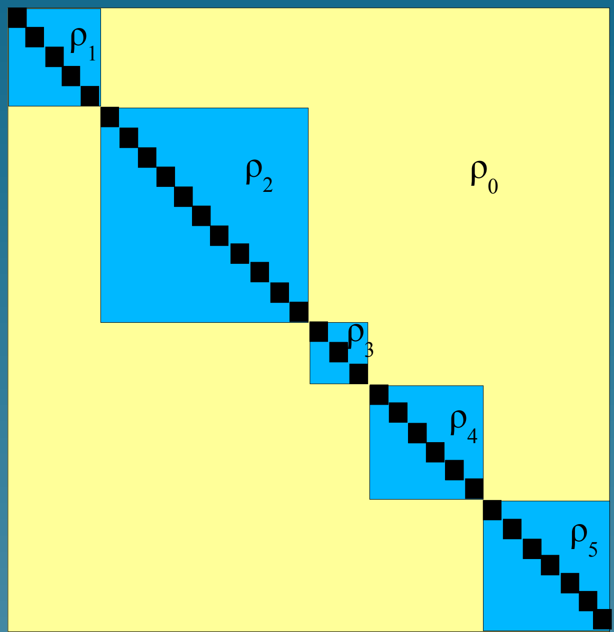
What can be done?

- In case we have some additional information about the covariance matrix, actually, the number of measurements may be less, than the number of elements in the covariance matrix, and still a reasonable optimization may be performed:

- Analysis shows, that typically there are a couple of large eigenvalues, and the rest is very close to the Random Matrix Model.

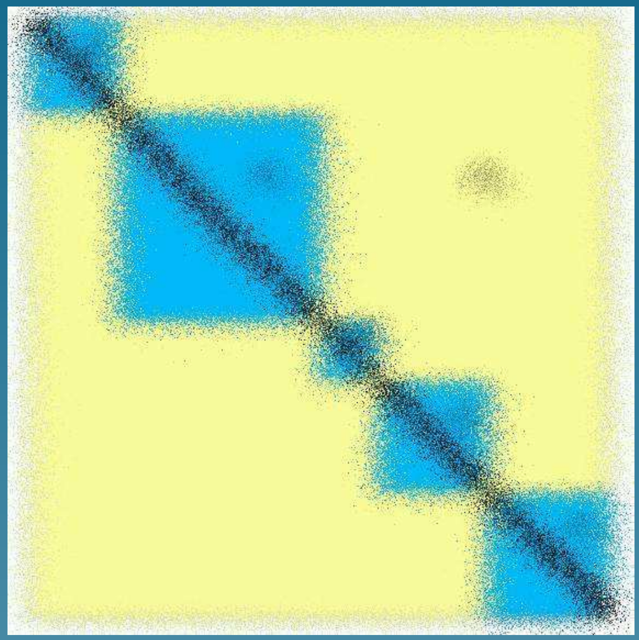
- The large eigenvalues can be associated with "sectors".
  - keep the entries associated with the sectors, and substitute the low lying part with the average eigenvalue.
  - use "cleaning" on the empirical covariance matrix assuming a certain number of sectors.

Sectorization



$\longrightarrow$

(Gaussian) noise



- Coupling between timeseries (Noh, Phys. Rev. **E61** (2000) 5981):
  - ϱ_0 : "market"
  - ϱ_i : sectors (energy, telecommunication, etc.)
  - There are M different degenerate, and M large eigenvalues for a matrix with M sectors.

- Noise dims the real structure.

Cleaning: How does it work on SP500?

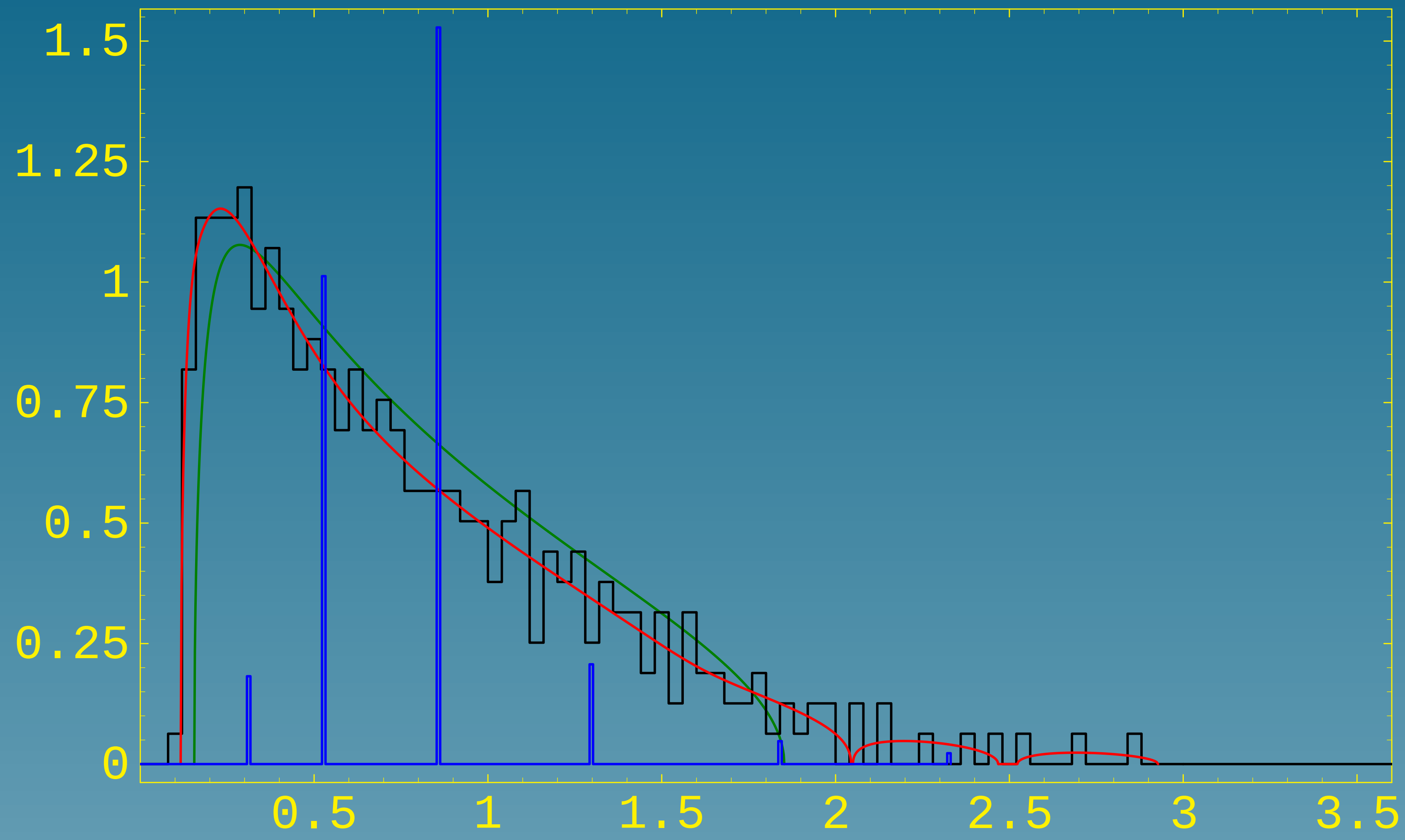
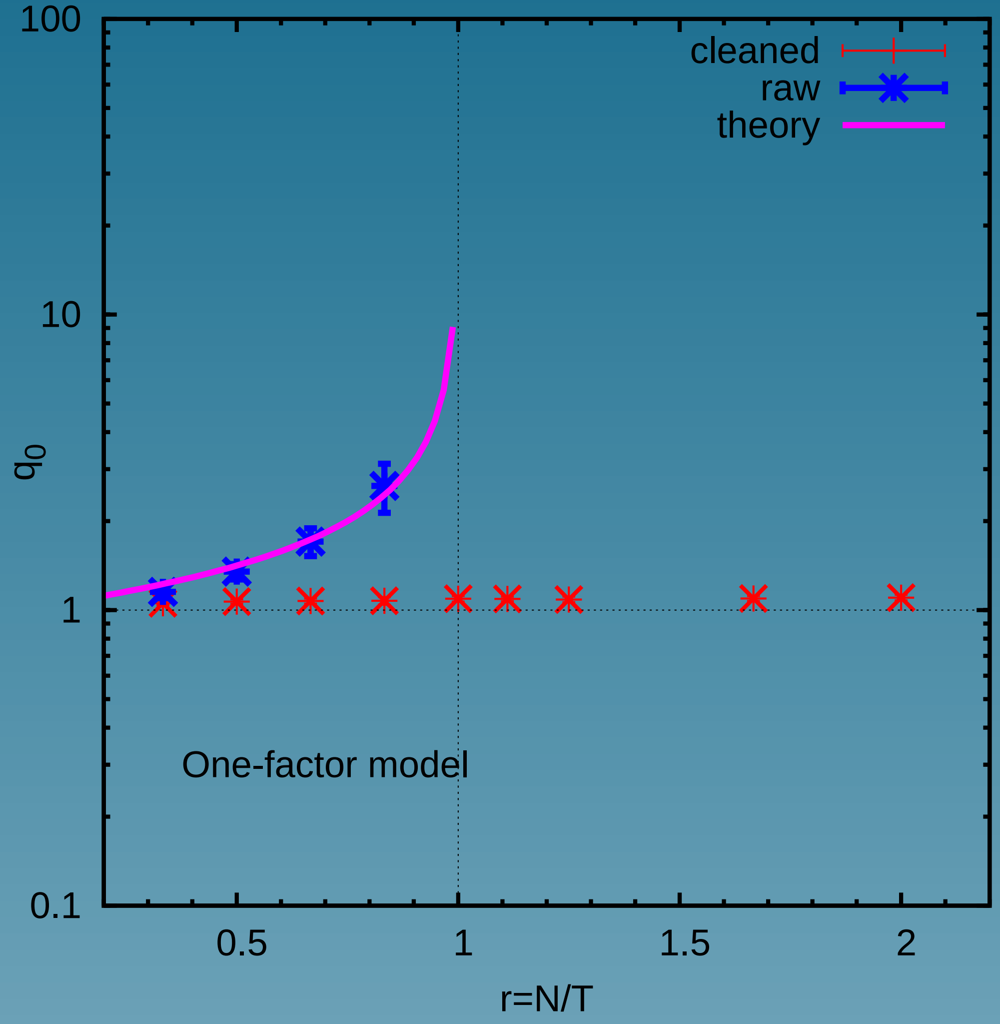


Figure 1: Approximation of the SP500 covariance matrix with a 6 “sector” assumption.

Benchmarking

- Check, how well the method works:
- Start with a know clean covariance matrix (assuming be the same as the correlation matrix).
- Add gaussian noise (generate finite price-change timeseries).
- reconstruct the best estimate of the clean covariance matrix using the cleaning technique.
- Compare the risk of the cleaned, empirical portfolio to the risk of the clean portfolio, using Markowitz (q_0).

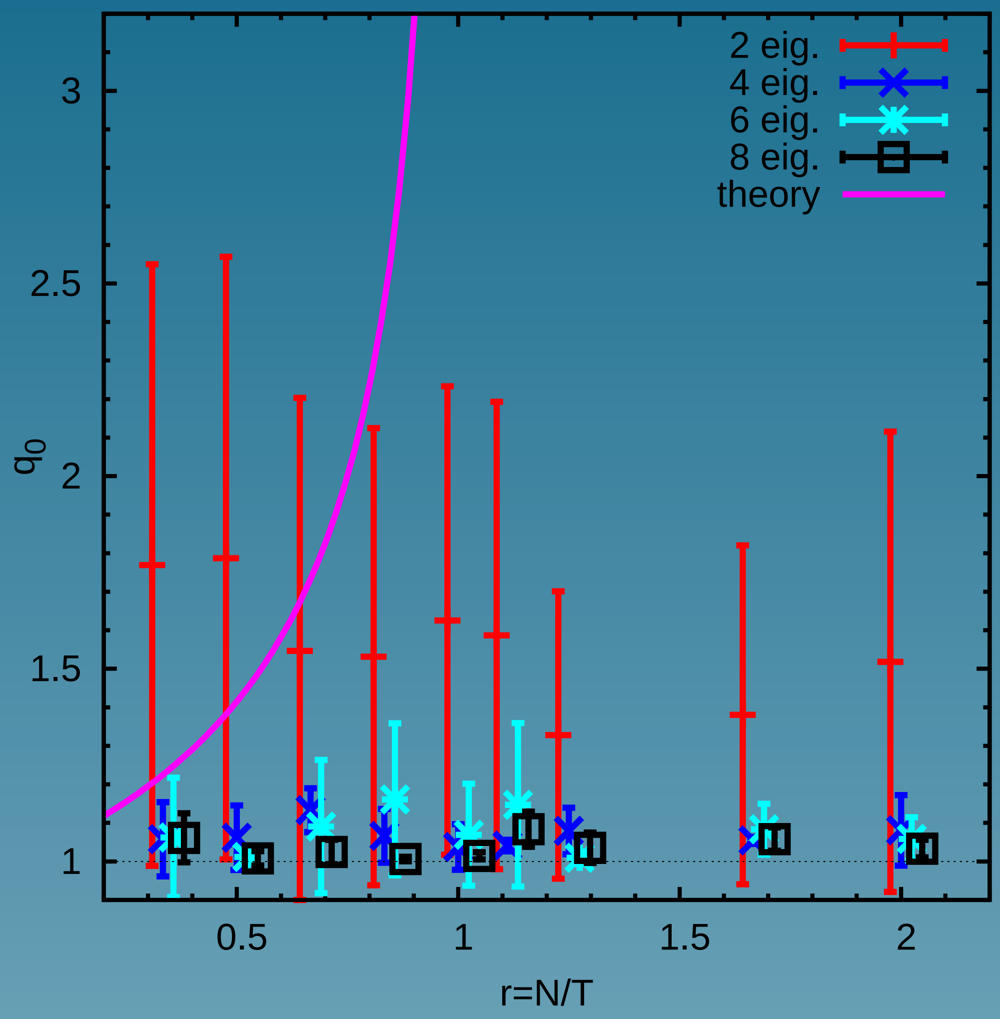
One factor model



Test of one-factor model for different values of r with fixed $N = 100$ and different number of sectors searched.

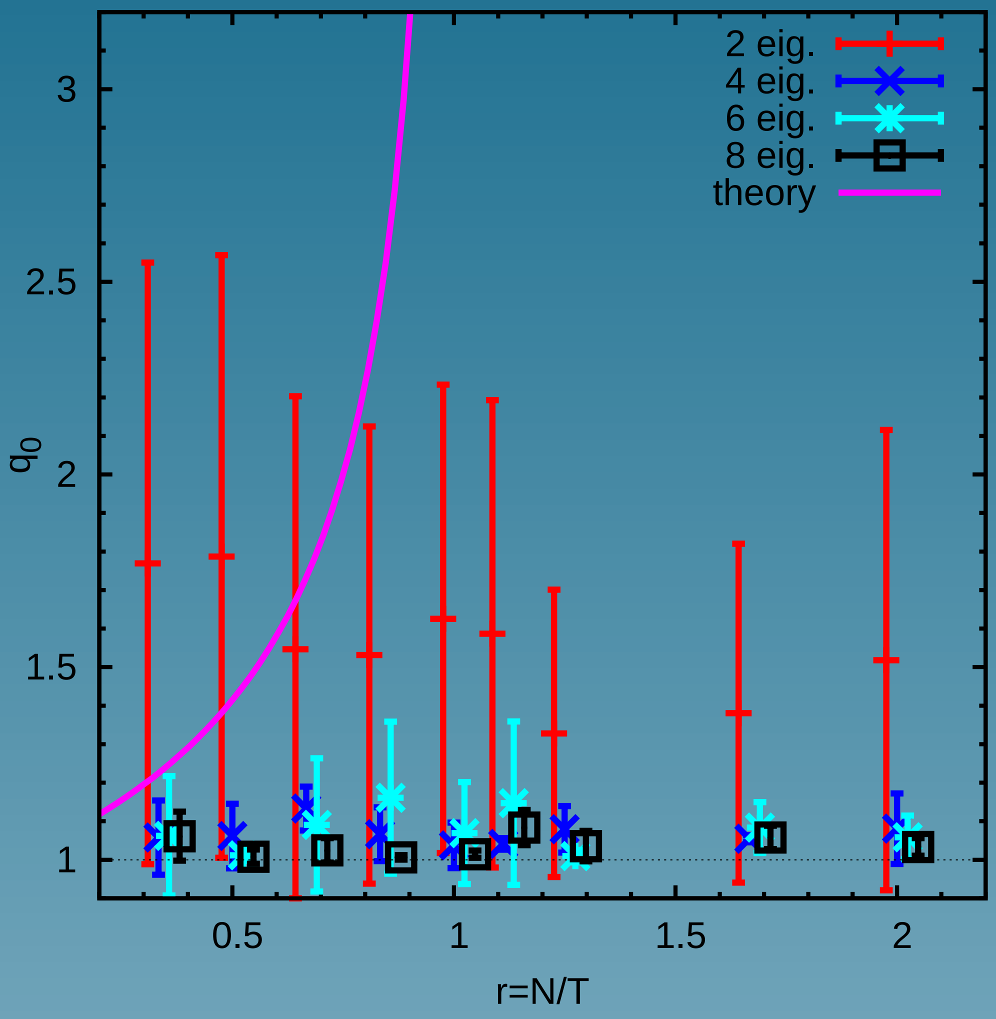
- perfect reconstruction
- does not depend on the number of sectors searched

Market-plus-sectors



Test of market-plus-sector model for different values of r with fixed $N = 100$ and different number of eigenvalues searched (2, 4, 6 and 8 eigenvalues).

Market plus-sectors



- searching for at least 4 eigenvalues gives a very good reconstruction
- no pronounced dependence on the number of eigenvalues searched
- Using less eigenvalues than necessary, worsens the situation: even the raw empirical matrix provides a better solution!

Summary

- Finite value of the ration r=N/T causes a huge problem in portfolio optimization.
- Cleaning is possible using a Random Matrix Model.
- Cleaning may be used to optimize a portfolio.
- The method is robust, i.e. in case the number of sectors is unknown, overestimating it does not cause any problem.
- Cleaning normally gives better results, than a simple RMM based spectrum decompositon.

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